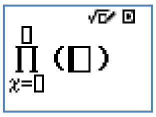
Dos problemas de logaritmos.

Problema 1:

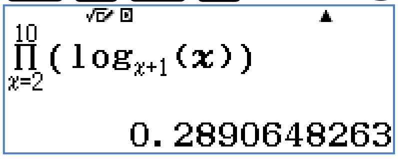
Comprobar que $\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{11} 10 = \log_{11} 2$.

Demostrar la igualdad y generalizar el resultado.

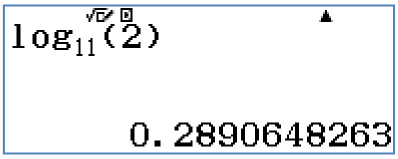
Solución:

Utilizaremos la función productos finitos [ALPHA] [x]  :

[ALPHA] [x] [log■□] [x] [+] [1] [▶] [x] [▶] [▶] [2] [▶] [1] [0] [=]



Calculemos $\log_{11} 2$:



Los dos resultados son iguales.

Demostración:

Aplicando el cambio de bases de logaritmos:

$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{11} 10 = \frac{\log_{11} 2}{\log_{11} 3} \cdot \frac{\log_{11} 3}{\log_{11} 4} \cdot \frac{\log_{11} 4}{\log_{11} 5} \cdot ..... \frac{\log_{11} 9}{\log_{11} 10} \cdot \log_{11} 10 = \log_{11} 2$$

Generalización:

$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{n+1} n = \log_{n+1} 2$, $n = 1, 2, 3, ...$

Problema 2

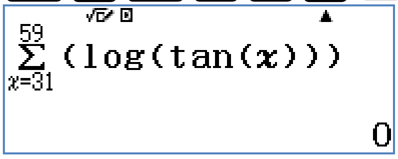
Comprobar que

$\log(\mathrm{tg}\ 31^{o})+\log(\mathrm{tg}\ 32^{o})+\log(\mathrm{tg}\ 33^{o})+........+\log(\mathrm{tg}\ 59^{o})=0$

Solución:

La calculadora tiene que estar en modo angular sexagesimal.

Utilizaremos la función sumas finitas:



Generalización:

$\log(\mathrm{tg}\ 1^{o})+\log(\mathrm{tg}\ 2^{o})+\log(\mathrm{tg}\ 3^{o})+........+\log(\mathrm{tg}\ 89^{o})=0$.

Demostración:

$\log(\mathrm{tg}\ 1^{o})+\log(\mathrm{tg}\ 2^{o})+\log(\mathrm{tg}\ 3^{o})+....+\log(\mathrm{tg}\ 89^{o})=$

$=\log(\mathrm{tg}\ 1^{o}\cdot\mathrm{tg}\ 2^{o}\cdot...\cdot\mathrm{tg}\ 44^{o}\cdot\mathrm{tg}\ 45^{o}\cdot\mathrm{tg}\ 46^{o}\cdot...\cdot\mathrm{tg}\ 88^{o}\cdot\mathrm{tg}\ 89^{o})=$.

$=\log(\mathrm{tg}\ 1^{o}\cdot\mathrm{tg}\ 2^{o}\cdot...\cdot\mathrm{tg}\ 44^{o}\cdot\mathrm{tg}\ 45^{o}\cdot\mathrm{ctg}\ 44^{o}\cdot...\cdot\mathrm{ctg}\ 2^{o}\cdot\mathrm{ctg}\ 1^{o})=\log(1)=0$